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F.M : 80	MATHEMATICS TEST 1	TIME: 3 HOURS

SECTION A (1 mark each)

1. The probability that a leap year has 53 Sundays is:
   A. 2/7 C. 1/53
   B. 1/7 D. 2/53
2. What is the angle between a and b if a.b = |a x b| ?
   A. $\pi/2$ C. $\pi/6$
   B. $\pi/4$ D. 0
3. If A = [1 0 4] and B = $\begin{bmatrix}2\\5\\6\end{bmatrix}$. Then AB is
   A. [24] C. [26]
   B. [25] D. [27]
4. The derivative of sin x w.r.t cos x is:
   A. Tan x C. –tan x
   B. Cot x D. –cot x
5. The value of $\int_{-\pi/2}^{\pi/2} \sin^7 x\, dx$ is :
   A. 0 C. 2
   B. 1 D. 3
6. What is the distance between the planes 2x + 2y – z + 2 = 0 and 4x + 4y – 2z + 5 = 0 ?
   A. 1/6 units C. 1/10 units
   B. 1/8 units D. ½ units
7. A random variable X, taking values 0,1,2 has following probability distribution for some number 'k'. P(X) = $\begin{cases} k, & \text{if } X = 0 \\ 2k & \text{if } X = 1 \\ 3k & \text{If } X = 2 \end{cases}$. Then the value of 'k' is:
   A. ¼ C. 1/2
   B. 1/6 D. 1/8
8. What is the total number of matrices of order 2 x 3 such that each entry is 0,1,2 ?
   A. 27 C. 343
   B. 125 D. 729
9. For what values of x is the function f(x) = $x^4 - \frac{4}{3}x^3$ is increasing?
   A. x ≤ 1 C. x ≠ 1
   B. x ≥ 1 D. x = 1
10. For the curve y = $(2x + 1)^3$, the rate of change of slope at x = 1 is:
    A. 18 C. 54
    B. 36 D. 72
11. The derivative of |x – 1| at x = -1 is
    A. 1 B. -1 C. 0 D. Not defined
12. A soldier fires three bullets on enemy. The probability that the enemy will be killed by one bullet is 0.7. What is the probability that the enemy is still alive?
    A. 0.027 B. 0.0027 C. 0.27 D. 0.00027
13. Three coins are tossed once. Find the probability of getting atleast one head.
    A. ¼ B. 1/8 C. ¾ D. 7/8
14. The maximum and minimum values of the function f(x) = Sin 2x + 5 are respectively given by
    A. 2 and -2 B. 1 and -1 C. 6 and 4 D. 5 and -5
15. The value of $\int_0^{\pi/2} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$ is
    A. 0 B. 2 C. 1 D. -1
16. The area of a parallelogram whose sides are given by $2\hat{\imath} - \hat{\jmath}$ and $\hat{\imath} + 5\hat{k}$ is
    A. √131 units B. 10 units C. √126 units D. 14 units
17. The degree and order of the differential equation $\sqrt{1 + \frac{dy}{dx}} = \left[\frac{d^2y}{dx^2}\right]^{1/3}$ respectively are:
    A. 1 and 2 B. 2 and 1 C. 2 and 2 D. 1 and 1

18. For what value of 'x' is the matrix $\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -4 \\ 3 & 4 & x+5 \end{bmatrix}$ is skew symmetric?

A. 5 B. -5 C. 0 D. 1/5

ASSERTION REASONING TYPE QUESTIONS (19 AND 20)

A. Both A and R are true and R is the correct explanation of A
B. Both A and R are true but R is NOT the correct explanation of A
C. A is true but R is false.
D. A is false but R is true.
E. Both A and R are false.

19.

Assertion (A) The value of the constant 'k' so that $f(x) = \begin{cases} kx^2, if\ x \le 2 \\ 3, if\ x > 2 \end{cases}$ is continuous at x =2 is k = 4/3.

Reason(R) A function f(x) is continuous at a point x= a of its domain if $\lim_{x \to a} f(x) = f(a)$

20.

Assertion (A) : Slope of the curve given as $y^2 = x$ at x = 1 not defined.

Reason (R) : Slope of the curve given as $y^2 = x$ at x = is $\pm\frac{1}{2}$.

SECTION B (2 marks each)

21. Form the differential equation of all the circles which touch the x – axis at the origin.
22. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ compute A^{-1} and show that $2A^{-1} = 9I - A$
23. Show that : $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$, $x \in [-1/2, ½]$
24. For what value of k is the function $f(x) = \begin{cases} 3x^2 - kx + 5 ; 0 \le x < 2 \\ 1 - 3x ; 2 \le x \le 3 \end{cases}$ is continuous.
25. Evaluate : $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx$

OR

Find the differential equation representing the family of curves $y = ae^{bx+5}$ where a and b are arbitrary constants.

SECTION C (3 marks each)

26. In a bolt factory machines A,B and C manufacture 60%, 30% and 10% of the total production respectively. 2%, 5% and 10% of the bolts produced by them are respectively defective. A bolt is picked from the lot and found to be defective. Find the probability that it was manufactured by the machine A.

OR

Two cards are drawn from a well shuffled pack of 52 cards. Find the mean and variance of the number of face cards obtained.

27. Show that a right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.
28. Solve the differential equation: $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x$, given that $y(0) = 0$.
29. Evaluate: $\int_1^2 \{|x-1| + |x-2| + |x-3|\}\, dx$
30. If f: R → R, g: R → R is given by f(x) = [x], g(x) = |x|, then find (fog)(-2/3) and (gof)(-2/3)

31. If x,y and z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$. Then show that xyz = -1

SECTION D (5 marks each)

32. Find the common area bounded by the circles: $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$

33. A company produces two types of lamps A and B. Both lamps go through a cutter and then a finisher. Lamp A requires 2 hours of cutter's time and 1 hour of finisher's time. Lamp B requires 1 hour of cutter's time and 2 hours of finisher's time. The cutter has 100 hours and the finisher has 80 hours of time each month. Profit on lamp A is Rs. 7 and that one lamp B is Rs. 13. Assuming that the company can sell all the lamps that it produces, how many lamps of each type should it produce to maximize the profit?

34. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

x – y + 2z = 1; 2y – 3z = 1; 3x – 2y + 4z = 2

OR

Solve by matrix method: (x ≠ 0, y≠ 0, z≠ 0)

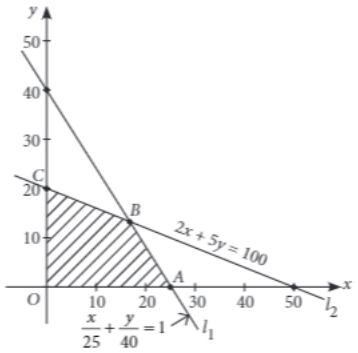
$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10; \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10; \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

35. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 5\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$

SECTION E (4 marks each) CASE BASED QUESTIONS

36. CASE STUDY 1:

Deepa rides her car at 25 km/hr. She has to spend ₹ 2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to ₹ 5 per km. She has ₹ 100 to spend on diesel. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr. The feasible region for the LPP is shown below :



(i) What is the point of intersection of line l_1 and l_2.

(a) $\left(\frac{40}{3}, \frac{50}{3}\right)$ (b) $\left(\frac{50}{3}, \frac{40}{3}\right)$ (c) $\left(\frac{-50}{3}, \frac{40}{3}\right)$ (d) $\left(\frac{-50}{3}, \frac{-40}{3}\right)$

(ii) The corner points of the feasible region shown in above graph are

(a) (0, 25), (20, 0), $\left(\frac{40}{3}, \frac{50}{3}\right)$ (b) (0, 0), (25, 0), (0, 20)

(c) (0, 0), $\left(\frac{40}{3}, \frac{50}{3}\right)$, (0, 20) (d) (0, 0), (25, 0), $\left(\frac{50}{3}, \frac{40}{3}\right)$, (0, 20)

(iii) If $Z = x + y$ be the objective function and max $Z = 30$. The maximum value occurs at point

(a) $\left(\frac{50}{3}, \frac{40}{3}\right)$ (b) (0, 0) (c) (25, 0) (d) (0, 20)

(iv) If $Z = 6x - 9y$ be the objective function, then maximum value of Z is

(a) –20 (b) 150 (c) 180 (d) 20

37. CASE STUDY 2

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association).

Let A be a 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.

(i) If ₹ 15000 is invested in bond X, then

(a) $A = \begin{matrix} X \\ Y \end{matrix}\begin{matrix}\overset{\text{Investment}}{\begin{bmatrix} 15000 \\ 20000 \end{bmatrix}}\end{matrix}$; $B = \begin{bmatrix} X & Y \\ 0.1 & 0.08 \end{bmatrix}$ Interest rate

(b) $A = \text{Investment}\begin{matrix}\overset{X \quad Y}{\begin{bmatrix} 15000 & 20000 \end{bmatrix}}\end{matrix}$; $B = \begin{matrix} X \\ Y \end{matrix}\begin{matrix}\overset{\text{Interest rate}}{\begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}}\end{matrix}$

(c) $A = \text{Investment}\begin{matrix}\overset{X \quad Y}{\begin{bmatrix} 20000 & 15000 \end{bmatrix}}\end{matrix}$; $B = \begin{matrix} X \\ Y \end{matrix}\begin{matrix}\overset{\text{Interest rate}}{\begin{bmatrix} 0.08 \\ 0.1 \end{bmatrix}}\end{matrix}$

(d) None of these

(ii) If ₹ 15000 is invested in bond X, then total amount of interest received on both bonds is

(a) ₹ 2000 (b) ₹ 2100 (c) ₹ 3100 (d) ₹ 4000

(iii) If the trust fund obtains an annual total interest of ₹ 3200, then the investment in two bonds is

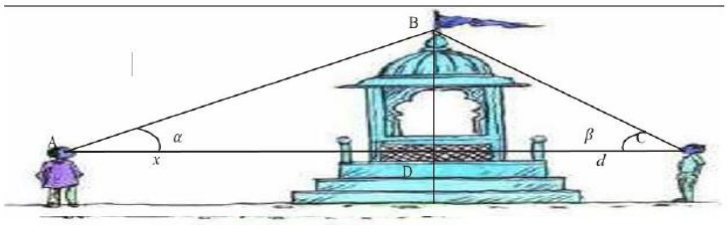
(a) ₹ 15000 in X, ₹ 20000 in Y (b) ₹ 17000 in X, ₹ 18000 in Y

(c) ₹ 20000 in X, ₹ 15000 in Y (d) ₹ 18000 in X, ₹ 17000 in Y

(iv) The total amount of interest received on both bonds is given by

(a) AB (b) $A'B$ (c) $B'A$ (d) none of these

38. CASE STUDY 3



Two men on either side of a temple of 30 meters high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters. Based on the above information answer the following:

i) Angle CAB =

a) $\text{Sin}^{-1}\, 2/3$ b) $\sin^{-1} ½$ c) $\sin^{-1} 2/7$ d) $\sin^{-1} 1$

ii) Angle BCA =

a) $\text{Tan}^{-1}\, .5$ b) $\tan^{-1} 2$ c) $\pi/6$ d) $\pi/3$

iii) Angle ABC =

a) $\pi/4$ b) $\pi/3$ c) $\pi/6$ *d)* $\pi/2$

iv) *Write the domain and range of* $\cos^{-1}x$