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F.M : 80	MOCK TEST 2	TIME: 3 HOURS

SECTION A (ONE MARK EACH)

1. Find a point on the curve y = $(x – 2)^2$. at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).
   (a) (3, 1) (b) (4, 1) (c) (6,1) (d) (5, 1)
2. The shortest distance between the lines x = y + 2 = 6z – 6 and x + 1 = 2y = -12z is
   a. ½ b. 2 c. 1 d. 3/2
3. Cos $(2\tan^{-1} 0.5)$ = ?
   a. 3/5 b. 4/5 c. 7/5 d. none of these
4. The vectors λi+j+2k,i+λj−k and 2i−j+λk are coplanar if
   (a) λ = -2 (b) λ = 0 (c) λ = 1 (d) λ = -1
5. The function f(x) = $\tan^{-1}$ (sin x + cos x) is an increasing function in
   a. $(\frac{\pi}{4}, \frac{\pi}{2})$ b. $(-\frac{\pi}{2}, \frac{\pi}{2})$ c. $(0, \frac{\pi}{2})$ d. none of these
6. The probability of a man hitting a target is 1/4. How many times must he fire so that the probability of his hitting the target at least once is greater than 23?
   (a) 4 (b) 3 (c) 2 (d) 1
7. Evaluate $\int_0^1 sin^{-1}(\frac{2x}{1+x^2})$
   a. $\frac{\pi}{2} - \log 2$ b. π c. $\frac{\pi}{4}$ d. $\frac{\pi}{2} + \log 2$
8. Matrix A and B will be inverse of each other only if
   (a) AB=BA (b) AB=BA=0 (c) AB=0, BA=I (d) AB=BA=I
9. If AB × AC = 2i−4j+4k, then the area of ΔABC is
   (a) 3 sq. units (b) 4 sq. units (c) 16 sq. units (d) 9 sq. units
10. The function f(x) = $x^5 – 5x^4 + 5x^3 – 1$ has
    (a) one minima and two maxima (c) two minima and two maxima
    (b) two minima and one maxima (d) one minima and one maxima
11. A die is thrown and card is selected a random from a deck of 52 playing cards. The probability of gettingan even number on the die and a spade card is
    (a) ½ (b) ¼ (c) 1/8 (d) 3/4
12. Find x, if $\begin{vmatrix} 1 & 2 & x \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$ is singular
    a. 1 b. 2 c. 3 d. 4
13. If f : R → R and g : R → R defined by f(x) = 2x + 3 and g(x) = $x^2 + 7$, then the value of x for which f(g(x)) = 25 is
    (a) ±1 (b) ±2 (c) ±3 (d) ±4
14. Find the height of a cylinder, which is open at the top, having a given surface area, greatest volume and of radius r.
    (a) r (b) 2r (c) r/2 (d) 3πr/2
15. The derivative of f(tan x) w.r.t. g(sec x) at x = π/4, where f'(1) = 2 and g'(√2) = 4, is
    (a) 1/√2 (b) √2 (c) 1 (d) 0
16. If y = $(1 + x)(1 + x^2)(1 + x^4)\ldots(1 + x^{2n})$, then the value of dy/dx at x = 0 is
    (a) 0 (b) -1 (c) 1 (d) None of these
17. The minimum value of Z = 3x + 5y subjected to constraints x + 3y ≥ 3, x + y ≥ 2, x, y ≥ 0 is:
    A. 5 B. 7 C. 10 D. 11
18. The solution of the DE $\frac{dy}{dx} = \frac{-2xy}{x^2+1}$ is
    a. $Y^2 (x +1) = C$ b. $y(x^2 + 1) = C$ c. $x^2 (y +1) = C$ d. None of these

Directions for assertion reasoning questions

(a) Both A and R are correct; R is the correct explanation of A.
(b) Both A and R are correct; R is not the correct explanation of A.

(c) A is correct; R is incorrect.
(d) R is correct; A is incorrect.

19. Assertion (A) : The tangent to the curve $\boldsymbol{y = x^3 - x^2 - x + 2}$ at (1,1) is parallel to the x- axis .
Reason (R): The slope of the tangent to the curve at (1,1) is zero.

20. Assertion (A) Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. R is an equivalence relation
Reason (R) Since R is reflexive, symmetric but R is not transitive.

SECTION B (TWO MARKS EACH)

21. Evaluate P(A ∪ B), if 2P (A) = P(B) = 5/13 and P(A/ B) = 2/5.
22. Given, $\int e^x (\tan x + 1) \sec x \, dx = e^x f(x) + C$. Write f(x) satisfying the condition.
23. If $\tan^{-1}x + \tan^{-1} y = \pi/4$; xy < 1, then write the value of x + y + xy
24. Let R is the equivalence relation in the set A = {0,1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a – b)}. Write the equivalence class [0]
25. Form the differential equation representing the family of curves $y = e^{2x}(a + bx)$, where 'a' and 'b' are arbitrary constants

OR

Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

SECTION C (THREE MARKS EACH)

26. Find the position vector of a point R, which divides the line joining two points P and Q whose position vectors are $2\vec{a} + \vec{b}$ and $\vec{a} - 8\vec{b}$ respectively, externally in the ratio 1 : 2. Also, show that P is the mid-point of line segment RQ
27. Find the area of the smaller region bounded by the ellipse $x^2/9+y^2/4 = 1$ and the line $x/\mathbf{3}$ +y/2 = 1
28. Evaluate : $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$
29. Prove that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$
30. If $x^y + y^x = ab$, find dy/dx

Or

If $y = x^x$, then prove that : $\frac{d^2y}{dx^2} - \frac{1}{y}(\frac{dy}{dx})^2 - \frac{y}{x} = 0$

31. Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

SECTION D (FIVE MARKS EACH)

32. By examining the chest X-ray, the probability that a person is diagnosed with TB when he is actually suffering from it, is 0.99. The probability that the doctor incorrectly diagnoses a person to be having TB, on the basis of X-ray reports, is 0.001. In a certain city, 1 in 1000 persons suffers from TB. A person is selected at random and is diagnosed to have TB. What is the chance that he actually has TB?
33. Using properties of determinants prove that:

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

34. Find the equations of the line passing through the points A(0, 6, -9) and B(-3, -6, 3). If D is the foot of the perpendicular drawn from a point C(7, 4, -1) on the line AB then find the coordinates of D and the equations of the line CD.
35. Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4, and y = 0 into three equal parts

OR

Prove that $\int (\sqrt{\tan x} + \sqrt{\cot x})dx = \sqrt{2}.\frac{\pi}{2}$ if Limits of the integral are 0 to π/4

SECTION E (FOUR MARKS EACH)

36. CASE STUDY 1

A JEE aspirant estimates that she will be successful with an 80 percent chance if she studies 10 hours per day, with a 60 percent chance if she studies 7 hours per day and with a 40 percent chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively.

a) The chance she will be successful is
   i) 0.28 ii)0.38 iii)0.48 iv)0.58
b) Given that she is successful, the chance she studied for 4 hours, is
   i) ½ ii) 7/12 iii) 2/3 iv) ¾
c) Given that she does not achieve success, the chance she studied for 4 hours, is
   i) 18/26 ii)19/26 iii)20/26 iv)21/26
d) The probability that she will be unsuccessful is
   i) 0.72 ii)0.62 iii) 0.52 iv)0.42

37. CASE STUDY 2

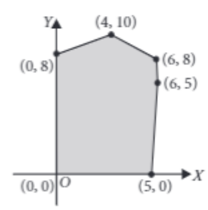
Let R be the feasible region (convex polygon) for a linear programming problem and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.
Based on the above information, answer the following questions.

(i) Objective function of a L.P.P. is
(a) a constant (b) a function to be optimised
(c) a relation between the variables (d) none of these

(ii) Which of the following statement is correct?
(a) Every LPP has at least one optimal solution.
(b) Every LPP has a unique optimal solution.
(c) If an LPP has two optimal solutions, then it has infinitely many solutions.
(d) None of these

(iii) In solving the LPP : "minimize $f = 6x + 10y$ subject to constraints $x \geq 6$, $y \geq 2$, $2x + y \geq 10$, $x \geq 0$, $y \geq 0$" redundant constraints are
(a) $x \geq 6, y \geq 2$ (b) $2x + y \geq 10, x \geq 0, y \geq 0$
(c) $x \geq 6$ (d) none of these

(iv) The feasible region for a LPP is shown shaded in the figure.
Let $Z = 3x - 4y$ be the objective function. Minimum of Z occurs at
(a) (0, 0)
(b) (0, 8)
(c) (5, 0)
(d) (4, 10)



38. CASE STUDY 3

A group of students of class XII visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Raj path (formerly called the Kingsway), is about 138 feet (42 metrs) in height.

A. They want to see the tower at an angle of $\sec^{-1} 2$. So, they want to know the distance where they should stand and hence find the distance.
B. What is the angle of elevation if they are standing at a distance of 42m away from the monument?
C. If the altitude of the Sun is at $\cos^{-1} 1/2$, then the height of the vertical tower that will cast a shadow of length 20 m is ___________
D. Domain of $\sin-1\ x$ is……..